Quantum computing with continuous time evolution

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Poster abstract for UCNC 2021. We will summarise our recent work on this promising alternative to standard digital quantum computing, that is more experimentally feasible in the short term.

Roadmaps for quantum computing usually plot a path from current state-of-the-art noisy quantum processors to large-scale fully fault-tolerant, error corrected digital quantum computers. The quantum equivalent of our digital silicon classical computer hardware is a logical goal, given the success and ubiquity of today's silicon CMOS technology. However, alternative models of quantum computing exist and promise to be useful within a shorter time-frame. One such alternative evolves the state of the quantum computer continuously in time, rather than using discrete gate operations to construct the computation.

Historically, this goes back to the original ideas for quantum computing itself: simulating quantum systems [1]. Using one quantum system to simulate the Hamiltonian dynamics of another quantum system was not given a digital setting until Lloyd [2]. Around the same time, continuous-time quantum walk computation [3, 4], adiabatic quantum computing [5], quantum annealing [6, 7], were proposed as models of computation using a continuous time evolution.

The discretisation into qubits used in the circuit model of quantum computing is not the only way to encode data into physical systems. Like their classical counterparts, analog quantum computers use continuous variables to encode the problem [8, 9], with a different universal set of discrete, unitary transformations to evolve the quantum state. A physical quantity that can be varied continuously, such as length, position, or momentum, is used to approximate a real number by making it proportional in size. However, a unary encoding is exponentially inefficient for classical data compared with a binary or digital encoding. This is not specific to quantum information. Using n bits we can represent 2^n different numbers, exponentially more than the number of bits. Analog encoding is proportional to the size of the number. A number twice as large doubles the size of the analog quantity representing it, whereas it requires only a single extra bit for binary encoding. Efficient computation therefore requires discrete classical data encoding, into bits or higher dimensional units. The same applies to quantum computing, where the most common choice will be qubits, two-state systems, although it doesn't need to be binary. Any reasonable choice of qudit of dimension d [10, 11] will do. There are implications for the complexity of the gate operations in the gate model for $d > 2$, but these don't necessarily carry over to continuous-time quantum computing, where the aim is to use the natural interactions present in the system.

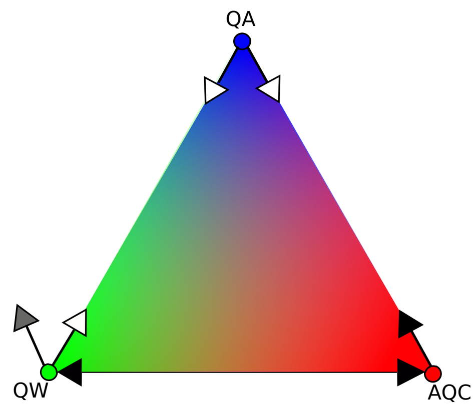

Figure 1. Relationships between different types of CTQC. This is the zero temperature plane, with added thermal noise providing a third dimension.

Digital (binary) data encoding with continuous time evolution leads us to consider adiabatic quantum computing, computing by continuous-time quantum walks, quantum annealing, and a range of special purpose quantum simulators, which all evolve the quantum state continuously in time. Quantum annealing and special purpose quantum simulators especially are candidates for early implementation, before the exacting demands of error-corrected digital quantum computing can be achieved. Quantum computing with continuous time evolution is also less thoroughly studied than the circuit model and measurement-based quantum computing. Given the importance for the first generation of useful quantum computers, this computational potential deserves more intensive development.

As well as unitary time evolution under a natural interaction Hamiltonian and/or a driving Hamiltonian, we can make use of more general transformations due to interactions with the environment. From a theoretical point of view, physically feasible non-unitary transformations can be modelled by enlarging the quantum state to include the environment: the global transformation will be unitary in this larger state space. Ancilla qubits that are not

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measured can perform this function in a quantum circuit. Here we will consider a more practical approach, in which the environment is explicitly engineered to provide particularly desirable properties, such as a low temperature bath.

Adiabatic quantum computing, quantum annealing, and computation by continuous-time quantum walk are all facets of the same computational model visualised in figure 1. The base of the triangle, from QW to AQC is pure unitary quantum evolution under the Hamiltonian

$$\hat{H}(t) = A(t)\hat{H}_0 + B(t)\hat{H}_p.$$

The difference between QW and AQC is in how $A(t)$ and $B(t)$ are specified. For QW they are constant throughout the time evolution, while for AQC there is a smooth transition of $A(0) = 1$ to $A(t_f) = 0$, while $B(0) = 0$ to $B(t_f) = 1$, for t_f the duration of the computation. The time dependence of $A(t)$ and $B(t)$ is optimised to keep the system close to the ground state. The black double-headed arrow on the base between QW and AQC indicates that this interpolation has been explored [12]. Hybrid strategies combining both QW and AQC have been found to be better than either alone, depending on the hardware characteristics.

Continuous-time evolution is naturally what interacting quantum systems do, left to themselves. There are ballistic models of quantum computation, and computation by quantum walk can be set up this way. However, most generally the time-evolution will be externally driven, by applying either global or local controls. We consider any physically reasonable controls, including coupling to ancilla systems, measurement, and unitary Hamiltonian time evolution. Continuous-time quantum computation can utilise any combination of

- constant Hamiltonian time evolution (quantum walk),
- slowly changing Hamiltonian evolution (adiabatic),
- quenching and arbitrarily time-varying Hamiltonians (quantum annealing),
- coupling to low temperature baths (open quantum system effects),
- different choices of basic Hamiltonians (quantum simulation),

in any proportions and sequence permitted by the hardware. This provides a wide range of options for solving specific problems efficiently. Note that "discret" gates are possible in some circumstances, e.g., a global phase inversion for spin-echo effects, since in practice they are applied using continuous-time control sequences.

Recent advances in techniques to control and optimise continuous-time quantum computing include Morley et al [12] showing how to interpolate between adiabatic quantum computing and quantum walks in the context of the search algorithm, allowing hybrid strategies to be employed to optimise the algorithm to the hardware characteristics. Callison et al [13] demostrated that quantum walks can successfully tackle hard optimisation problems using a strategy of repeating many short runs. And in Callison et al [14] theoretical foundations are laid down to support efficient heuristics for setting parameters in continuous-time quantum algorithms.

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